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Report on the I.U.T.A.M. symposium on concentrated vortex motions in fluids

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A symposium on the subject 'Concentrated vortex motions in fluids' was held at the University of Michigan, Ann Arbor, Michigan, from 6 to 11 July 1964. The symposium was organized by the International Union for Theoretical and Applied Mechanics, and participation was restricted to about 150 people invited on the basis of their active interest in the subject. The author was the secretary of the international committee which planned the meeting, and has prepared the following account of the scientific developments of the symposium in order to make them widely available.† There will be no other publication of the proceedings of the symposium. Further details of the work described at the symposium, and of related previous work, are given in the references quoted.

Introduction

The beauty and vigour of whirling and swirling vortex motions in water and air have fascinated man from the earliest days. Vortices may have inspired Mediterranean artists and craftsmen well over 3000 years ago to their chains of spiral ornamentations. Vortices obsessed Leonardo da Vinci who regarded the violent revolving movements as the only ultimate alternative to the powers striving to maintain a straight course. Today, vortex motions are still described as the sinews and muscles of fluid motions and the classical subject of research finds ever new applications: From the structure of turbulent motions to tea-cup and bath-tub whirls; to vortex tubes with their possible applications in computer machinery, in nuclear reactors and in other energy-conversion schemes;

† *Editors' note.* This novel method of reporting the proceedings of a symposium to the scientific community at large deserves consideration by the organizers of other meetings. There are obvious objections to the common plan of publishing in one volume the papers presented at a meeting: being an isolated volume it is not certain to be acquired by libraries; the papers are published at a time and under conditions not known in advance to authors; and papers are not usually subjected to the scrutiny of referees before publication. However, some kind of widely available report on an important meeting seems to be desirable, particularly when participation is by invitation only. Dr Küchemann's detailed account of the papers presented and the discussion that followed them should enable readers to see whether there were any developments at the Michigan Symposium of direct interest to them, and any such developments can be followed up with the aid of the very complete list of references at the end of the report.

The Editors will be glad to consider other similar reports of conferences concerned with some aspect of fluid mechanics for publication in the *Journal.*

to aeroplanes and other lifting bodies; to wakes behind buildings, moving objects, and oscillating cables; to dust devils, hurricanes and cyclones in the atmosphere; and to vortex phenomena in cosmic systems. The present survey of the matters presented at the symposium is necessarily very brief, but it may, nevertheless, reveal the individual diversity and the underlying unity of the subject.

On the whole, the fluid motions discussed involve mainly inertia, pressure, and viscous forces and sometimes buoyancy. The presentation of the subject matter follows roughly the groups into which the papers were divided at the symposium. The first section is concerned with the formation of coherent vortex sheets, as a result of flow separation from solid bodies, and their rolling-up. A discussion of the structure of the rolled-up cores and of the various possible deviations from smooth cores follows. The next section deals with the occurrence of columnar vortices in rotating fluid systems, especially in the atmosphere, where the mechanisms by which vorticity is generated and concentrated present major problems. Finally, vortex wakes behind bodies are discussed, where the classical subject of the flow past circular cylinders and Kármán vortex streets plays a prominent part. This grouping is more a matter of convenience than a reflexion of physical differences; the many strong inter-relations between the various individual problems will make this clear.

The formation of coherent vortex sheets

A steady flow which exhibits some of the characteristic features of the continuous formation of coherent vortex sheets very clearly is that past a three-dimensional conical body with flow separation along rays from the apex. The flow near the apex of a thin flat delta wing at an angle of incidence, first described by Roy (1952), is the prominent example. In a model for an inviscid flow, the shape of the infinitely thin vortex sheets, which are stream surfaces and sustain no pressure difference, is taken as conical and, in the basic case, the sheets from the two sides coil up above the wing in endless spirals. The inner parts of each form a core in which much of the vorticity is concentrated. The results from existing theories, surveyed by Legendre* in an invited review lecture, do not agree satisfactorily with observation and several improved, albeit incomplete, treatments were presented, each approaching the problem from a different aspect.

The earlier quantitative results were all obtained from treatments in which the assumption of slenderness was made for the whole flow field, implying that changes in the direction of the mainstream of the velocity component in this direction could be regarded as small compared with crosswise velocity gradients. Smith* and Maskell* retained this assumption and considered that it was important to fulfil the boundary conditions more accurately than before along the whole of the sheet from the leading edge right into the centre of the core. With the help of an electronic computer and an appropriate iterative procedure

* An asterisk to a name indicates that the work concerned was described in a lecture at the Symposium.

for solving the highly non-linear equations involved, Smith obtained interim results which showed a considerable improvement on those given earlier by Mangler & Smith (1959) and are capable of further improvement. The asymptotic form of the spiral near its centre as found by Mangler & Smith was improved by Maskell. He found that, although the dominant contribution to the circumferential velocity near the centre was locally self-induced, the local contribution to the radial velocity was only of the same order as that from those parts of the vortex sheets remote from the spiral core. This resulted in a modification of the shape for which there was some experimental support in the work of Earnshaw (1962). However, Maskell's core solution cannot be uniformly valid because it implies an infinite axial velocity at the core centre, which violates the slenderness assumption made in its derivation. It is clear that a stage has been reached in this work where further extensions should lead to solutions which are uniformly valid in the core and apply to real flows where the changes in axial velocities are no longer small. As was mentioned in the discussion, this might come about through the recent work of Mangler & Weber (1964) where an asymptotic solution for incompressible flow is given. Only then can an allowance be made for a radial inflow of fluid into the core region, to be converted into axial flow, and hence for the strong interaction between axial and swirl velocities, which is one of the key features of vortex motions.

Legendre* noted the difficulties arising in the slender solutions and proposed to avoid them by dropping the slenderness assumption and treating a conical approximation to incompressible flow. This inevitably complicated the analysis, but he was able to present preliminary calculations relating to an isolated vortex model, which differed from comparable slender-theory calculations but tended to show only a small dependence on aspect ratio. Instead of treating the three-dimensional incompressible flow in full, Roy* represented the changes in streamwise velocity from one cross-flow plane to the next by distributions of sources and sinks and considered the problem in the case in which these are concentrated on the vortex sheet and in the core.

Some of the features of these flows come out more clearly when a thin jet sheet is ejected along the leading edge. The curved jet sheet with a finite momentum in some cross-wise direction is now capable of sustaining a pressure difference across it and, consequently, there is a load along the leading edge. This in turn changes the otherwise unfavourable pressure field over part of the upper surface and even relatively small blowing rates eliminate the secondary separation there. On the other hand, matters are complicated by the need to investigate also the fluid flow in the jet, which may not reach the core region. Lilley* discussed how the theory of Mangler & Smith (1959) might be extended to this case and demonstrated the strong and beneficial effects of such blowing by means of experimental results obtained by Alexander (1963).

Alexander* presented some two-dimensional time-dependent flows with spiral vortex sheets, governed by similarity relations and consistent with wedge flow at infinity. These extended the similarity solutions of Prandtl (1922) to many-branched sheets and, in the case of Prandtl's single-branched sheet, changed the range of similarity parameters in which solutions were found.

Dráský* also discussed two-dimensional flows with vortex sheets; but, by the inclusion of an isolated vortex-sink at the spiral centre, obtained a time-dependence different from Prandtl's and a concentration of vorticity with time towards the centre. All these spirals were equi-angular and thus differed essentially from the shapes found for conical flows.

Little attention has so far been paid to the behaviour of the flow near the separation line. No particular difficulties arise if separation takes place from a sharp edge of a body at which the included angle of the edge is zero, as in the flat-plate delta wings discussed above; while the general characteristics in viscous flows have been described earlier by Lighthill (1963). Attention has previously been drawn by Rott (1956) to the difficulties associated with separation from the edge of a wedge of non-zero included angle. Kraemer* reported on a first attempt to arrive at consistent mathematical models for steady three-dimensional inviscid potential flows involving separation from smooth surfaces and found a particular solution involving vorticity shedding, where the vortex sheet left the surface tangentially. However, future work may show that less restrictive solutions can be obtained if the shear layer along the body surface is taken into account, or if the flow is allowed to fluctuate with time.

Much of the experimental work described at the symposium was devoted to the study of the shedding of vortex sheets from particular bodies of various shapes and to the effects these sheets have. This work made use of an impressive and ingenious variety of means for visualizing the flows, from the introduction of particles such as smoke or hydrogen bubbles into the stream to the observations of clouds from the satellite Tiros; it led to photographs and films of often remarkable beauty. Vortex motions are, undoubtedly, photogenic. The objects shedding vortices ranged from islands to freely falling disks, with spheres and circular cylinders receiving most of the attention. The latter will be discussed in more detail below. Relevant work was reported by Rainbird & Crabbe* on inclined circular cones; by Valensi* on the helical vortices behind propellers and helicopter blades and on the wakes of chimneys and of ship superstructures; by Lakshminarayana & Horlock* on the leakage vortices in cascades of aerofoils, which are so important in the design of turbomachinery; by Lockwood Taylor* on vortex motions round tip tanks; and by Fink* on the deformation of vortex sheets above oscillating delta wings. Sutton* drew attention to a succession of separation lines on a wall upstream of an obstacle, giving rise to an array of coiled vortex sheets, each turning downstream around the sides of the obstacle.

Most of the cases discussed so far have in common that each of the resulting vortex sheets coils up along its free edge and forms only one core of concentrated vorticity and very little is as yet known about how this fundamental mode may be supplemented by others where a series of cores appears along the sheet. One mechanism which leads to this may be thought of in all those cases where two vortex sheets are fed continually from two edges and where the first two edge cores (and later on the subsequent cores) grow, either spatially downstream in a three-dimensional flow or with time, but not necessarily at the same rate. A certain amount of fluid is then drawn into each of these cores, which is divided from the fluid in the mainstream that is not drawn into the cores by stream surfaces that generally intersect the solid body from which the sheets spring in so-called attachment lines. An attachment line may run away from a separation line, in space or in time, and unless the body widens sufficiently, the attachment line of a stronger or closer sheet may intersect the separation line of a second sheet on the other side. This then causes an irregularity in the feeding of the second sheet and generally appears to lead to the formation of at least one other rolled-up core along the second sheet which in turn grows and, being closer to the surface, pushes the attachment line back to the other side, where a similar process may be initiated in due course. This possible periodicity in the formation of cores may thus have a very simple cause. Flows of this kind have been demonstrated in earlier experimental work by Maltby & Peckham (1956). The mechanism may operate even in situations where for reasons of symmetry a periodic shedding of vortex cores is not expected. Thus a steady three-dimensional flow with spatial periodicity along the body, or a flow which is periodic in space and time, may be observed rather than the expected two-dimensional flow. This may occur especially in the case of circular cylinders across a stream, where the positions of the separation lines need not be symmetrical but may change with time all along the span of the cylinder and thus encourage intersections between attachment and separation lines. The resulting flow may have the appearance of a Kármán vortex street; in particular, the vortex cores may be inclined or slanted relative to the body, and this may explain observations made by Roshko (1961), Wille* and others.

Whereas this big-scale periodicity has its origin in the flow near the body itself, other small-scale periodic formations of vortex cores along a vortex sheet may be due to an instability of the sheet itself. Phenomena of this kind on the boundaries of jets were described by Wille*, and Pierce (1961) has demonstrated for the case of a plate moving normal to itself in still air or water how an array of small-scale vortex cores can occur simultaneously with and be superimposed on a big-scale flow with one large core of the type already described. Betchov* showed that even single vortex filaments could assume helicoidal shapes which in turn were shown to be unstable. The basic problem of the Helmholtz instability of a plane vortex sheet was treated very thoroughly by van de Vooren* by means of an electronic computer. Whether the sheet was represented by a large number of potential line vortices or by a continuous distribution of vorticity, an initially smooth periodic deformation of its shape was shown to grow with time. The sheet distorted near certain points, tending towards spiral forms. Near these, concentrations of vortex elements were found, balanced by a reduction in their density elsewhere, in conformity with earlier results by Abernathy & Kronauer (1962). Similar results were shown also by Michalke* for a free shear layer of non-vanishing thickness subjected to periodic disturbances. Although the rolling-up process never leads to isolated line vortices, the vorticity distribution within the spiral cores must have the same effect on the far field as single-line vortices at their centres. No connexion has as yet been established between the flow near these cores and the flows discussed by Alexander* and Dráský*.

The structure of smooth vortex cores

The mathematical model used so far of concentrating vorticity on to a tightly wound spiral surface has, of course, no strict counterpart in real flows, although measurements of the velocity field in such cores have shown up one or two of the outer turns of the spiral. Further inwards, viscous diffusion smears out any trace of such discontinuities and an approximately axisymmetric swirling flow with distributed vorticity appears. Recently, Mangler & Weber (1964) have been able to show that, to a first approximation, mean velocity and pressure at a point in the inviscid flow incorporating a spiral vortex sheet are the same as those in the inviscid rotating core flow with distributed vorticity, as calculated by Hall (1961), in accordance with an earlier conjecture by Betz (1950). It may be said, therefore, that the two flow models are equivalent and, in particular, that the hypothetical diffusion process does not change the mean velocity components and the pressure field to a first order.

The swirling flow with distributed vorticity appears in many applications; it exists not only in the cores of rolled-up vortex sheets; it can also be produced by the conversion of an initial or boundary rotation into uniform rotary motion, as in vortex tubes, in bath-tub vortices, and in various atmospheric phenomena. The most striking characteristics of these flows are that they can convect fluid towards a centre and so oppose the effects of diffusion on, for example, the vorticity; that in them the flow can be predominantly swirling away from the axis and predominantly axial near it; and that they can separate different media and create gradients of total temperature. In an invited survey lecture Hall* surveyed this wide field.

These spiralling flows are in general considered to exist within certain boundaries and the conditions on these boundaries, such as the inflow and swirl components of the velocity, may together with some initial conditions on some upstream cross-section determine them. Thus the flows may be thought of either as embedded in external flows containing vortex sheets or as flows in containers which may have porous, slotted or rotating walls. Axisymmetric flows have received special attention and particular shapes of the boundary, such as cylindrical, quasi-cylindrical or slender, or conical shapes, allow various approximate solutions to be obtained. A good many, exhibiting certain common features, are now available. In many cases, viscous forces can be ignored in these flows, provided the vorticity is taken into account and a narrow inner viscous core, where the solutions are of the boundary-layer type, is admitted. The outer core structure is then determined by inertia and pressure forces alone. The key question again concerns the interaction between axial and swirl velocities, and the balance between pressure gradients and centrifugal forces often leads to very low pressures and high axial velocities near the centre. The flow there is very sensitive to changes along the outer boundary. For instance, pressure changes along the boundary may be considerably magnified near the axis. Unless the axial variations are in some sense small, when the flow has been described as quasi-cylindrical, the problem is virtually intractable and only idealized models admitting linearized solutions or similarity solutions or expansions based on similarity solutions have been considered.

The many aspects of the problems posed by these motions and the large variety of ways in which they can be produced are reflected in the great number of contributions. Hummel* presented experimental results for cores of rolled-up vortex sheets, which confirmed the inviscid core solutions of Hall (1961) and exhibited the very low pressures and high axial velocities which are typical of cores embedded in a nearly conical external flow with continuous feeding of vorticity into the core. In the essentially non-conical inner viscous core, however, the axial velocity was found to increase and the pressure to fall in a streamwise direction. Brown & Stewartson* presented an extension of Hall's theory to include the effects of compressibility and found that the only solutions acceptable on physical grounds have vanishing density on the axis. In general, compressibility reduces the variations across the core; on the axis, for example, the axial velocity was found to be finite and the circumferential velocity zero, in marked contrast to the infinite values for incompressible flow. Merzkirch* treated a corresponding two-dimensional time-dependent flow as it occurs when a shock wave passes over a vertical wedge and produces a vortex sheet springing from the apex of the wedge. An outer inviscid core solution had been obtained by Howard & Matthews (1956), and similarity solutions were constructed for the inner viscous core. A sink term was included in the equation of continuity; its physical significance is not yet clear, even though Merzkirch showed that fluid was brought continually from the outer to the inner core. The calculated density distribution was found to be in good agreement with experimental values. The result of an experimental investigation of the passage of a plane shock over a two-dimensional aerofoil and of its interaction with the starting vortex were reported by Dosanjh* with particular reference to the acoustic wave emitted as the vortex core is deformed by the shock wave.

When, in a three-dimensional flow, vorticity is no longer fed into the core, as in trailing vortices some distance behind wings, the core is quasi-cylindrical and the pressure rises in the stream direction. Sufficiently far downstream both the defect of axial velocity and the tangential velocity will be small compared with the free-stream velocity. Such flows were treated by Batchelor* who specifically accounted for the small but generally significant axial gradient of pressure through which the decay of swirling motion far downstream effects a reduction of the axial velocity. In this way, the vortex drag of lifting wings was shown to be gradually manifested as an ordinary wake. It became clear from Batchelor's work that the earlier work of Newman (1959), in which the axial pressure gradient is taken to be negligible, is limited to cases where the vortex drag is small compared with the profile drag. Again for trailing vortices far downstream of wings, Glauert* examined the gradual diffusion of a vortex doublet and contrasted the asymptotic behaviour with that of a vortex ring. McCormick* described the development of techniques for studying vortex motions in flight; in this context, the extensive data obtained by Rose & Dee (1963) should be mentioned.

Viscous effects in vortex motions driven by an inward radial convection of angular momentum were examined in more detail by Lewellen* who described exact solutions of the Navier–Stokes equations and presented some nearly exact solutions, obtained either by a general expansion of the equations of motion for large swirl, i.e. small Rossby number, or by linearizing the equations for perturbations about known flows. By departing from exact similarity, some freedom to satisfy boundary conditions was gained. Lewellen found that in flows dominated by rotation the fluid motion is forced to be two-dimensional except for thin shear regions where all necessary adjustments imposed by the boundary conditions are made. Gartshore* used momentum integral methods to calculate viscous laminar cores imbedded in an irrotational external stream. Ross* reported on experiments in vortex chambers where an intriguing variety of possible flow patterns was discovered. He studied especially the turbulence level in vortex chambers and determined an effective turbulent-to-laminar shear ratio ranging from 10 to 10^3. Deissler* considered core flows which are subjected to sudden changes of the swirl velocity, principally at the outer boundary, and obtained some numerical properties of the time-dependent growth and decay of vortex cores. Finally, Craya* described his investigation of similarity solutions for swirling jets in still air, in which he considered turbulent flow by making full use of the concept of eddy viscosity.

Various deviations from smooth core flows

Even though the smooth and regular core flows described so far are stable in many respects and often the preferred type of flow, there are many circumstances in which they cannot exist. Three main causes may be distinguished at the present stage, which make the existence of the basic swirling motion impossible:

(1) A longitudinal pressure field may be self-induced or impressed upon the core, either by an external flow or by the shape of the walls of a vortex tube, and this may bring the highly responsive flow near the centre of the core to rest (in a sense, this corresponds to the appearance of a separation point in the development of a two-dimensional boundary layer along a wall).

(2) The swirling flow may be unstable with regard to annular or spiral disturbances; in particular, it may become so if it encounters an adverse pressure gradient in its course.

(3) In the presence of an end wall or a pair of end walls, in a vortex tube or in the atmosphere, the development of the boundary layer along the wall or walls may affect the swirling flow; in particular, flow separation along these walls may occur.

In all these cases, a distinction may be made between investigations concerned with the events leading up to the 'non-existence' condition and those concerned with the events which actually happen afterwards. A stage has been reached in this work where an astounding variety of possible flow patterns begins to appear.

Flows of the first kind, in which stagnation points appear, were demonstrated by Ranz* and by Donaldson & Snedeker*. In both cases, swirl was imparted to fluid passing out of a duct. They found that, if the swirl velocity at the boundary was large enough compared with the axial velocity, the flow was brought to rest at some point on the axis and a free stagnation point appeared in front of a large egg-shaped bubble bounding fluid which itself rotated and contained distinct backflow components. The flow could be quite steady and became oscillatory only near critical conditions. The phenomenon may be associated with the rise in pressure from a low value inside the tube (at large swirls) to the value outside the open end; Ranz reproduced it also in an enclosed duct flow with widening cross-sections. Donaldson & Snedeker regarded the phenomenon as a stage in the transition from one-celled to two-celled flow with reversed flow direction near the axis all along the tube.

Reports on theoretical investigations of unstable flows of the second kind were given by Ludwieg* and Jones*. Axisymmetric and spiral disturbances were considered and Ludwieg presented stability criteria for the core flow of Hall (1961) and predicted instability to small disturbances if the pitch angle of the helical streamlines became too steep and hence the Rossby number too small. Similar instabilities occur in swirling flows in annular ducts and earlier experiments by Ludwieg (1960) have shown up spiral vortices which are closely related to the well-known Taylor vortices in rotating flows between two cylinders. They are also related to the development of Görtler vortices in a boundary layer; these were demonstrated by Čolak-Antić*. For free cores in an external stream, Ludwieg* considered further some non-linear effects which should occur beyond the stability boundary and Hummel* found similar phenomena in wind-tunnel experiments. Jones* suggested another form of spiral instability of the core to explain the water-tunnel tests which he described. The instability criteria may, therefore, be regarded as genuine limits beyond which the smooth core flow of Hall cannot exist. Instabilities were also observed in vortex tube flows and described by Keyes*. They could be shown up clearly by the behaviour of dye traces. With an electrolytic conductor as the working fluid, Keyes found that an applied magnetic field could damp out the disturbances which had grown and that it could stabilize the flow again.

The various theories discussed so far cannot describe how the flow develops further after a free stagnation point or an instability has appeared. Only in the case with a large bubble has a reasonably steady flow been observed (at least over the front part of the bubble), as an alternative to the steady smooth core flow. In all other cases, the alternative flow, which occurs when the smooth flow cannot exist, is of a highly unsteady nature and the spectacular phenomena exhibited in such flows are often described in a general way as vortex breakdown. Hall*, Lambourne*, Hummel*, Gartshore*, Reynolds*, and Weske* all gave evidence of some aspects of such dual flow régimes in whirling cores. Characteristically, changes in the flow field were introduced such that the previously tightly rolled-up smooth core could no longer hold together and burst, giving rise to what often appeared to be a considerable dissipation of energy.

The only theory of this basic phenomenon in vortex motions, which is concerned with different possible flow régimes, was presented by Benjamin* who described non-smooth, undular, flows with finite amplitude waves and suggested that other flows associated with considerable energy dissipation may be more violent members of the same family. In Benjamin's theory vortex breakdown is a finite transition between two dynamically conjugate states of flow, in analogy to the hydraulic jump in open-channel flow. He showed by variational methods that this is possible for axisymmetric parallel flow in a cylinder and that, for a fluid of prescribed total pressure and circulation, there can be more than one flow which satisfies the equation. Associated with each of these conjugate flows is a momentum flux or flow force. Since the values of the momentum flux differ, a jump from one flow to its conjugate is possible only if the difference in flux can be made up by the superposition of a standing wave. Benjamin proved that, given a smooth flow which cannot support standing waves, any conjugate flow must have an excess of momentum flux, and at the same time must be undular and support standing waves. Thus, since the superposition of a standing wave reduces the momentum flux, momentum can indeed be conserved in the jump. The proposal, therefore, was that breakdown is a jump from a smooth state, which cannot support standing waves, to an undular state which can. No swirl at all represents the extreme of the smooth state, and with increasing swirl and decreasing Rossby number the state approaches the critical. Smooth cores may be brought towards the critical condition by a superimposed longitudinal adverse pressure gradient which also happens to encourage the appearance of a stagnation point on the axis and the occurrence of a spiral instability.

Binnie* demonstrated a sequence of such flows in a swirling stream of water with a free surface discharged from a reservoir down a vertical pipe: first the weak transition of a flow which just remained smooth to a slightly undular flow, with waves and small energy dissipation, and finally the strong transition to a highly undular flow, with intense turbulence and large energy dissipation in the jump. Somewhat similar observations of flows without free surfaces were reported by Reynolds* and Weske*. For vortex cores in gases, the experimental evidence presented by Lambourne* and others indicated that the resulting non-smooth flow may take the form of a succession of axisymmetric bubbles, or of one bubble with highly turbulent flow at its rear end, or of a distinctive highly turbulent spiralling flow. This is where the matter rests for the moment and many questions remain open.

Flows of the third kind appear in what was described as canned vortices, i.e. in tubes with end walls. There is usually a strong interaction between the basic swirling flow and the three-dimensional boundary layer along such a wall. Rott*, reviewing first earlier work by Boedewadt (1940) and Stewartson (1957) and more recent numerical work by Anderson (1961), Mack (1962), and King (1963), presented an iterative analytical procedure for the solution of the problem of the boundary layer. Rosenzweig* gave experimental results for the overall flow and presented a theory for this flow which included the interaction between the vortex and the boundary layer. Anderson* gave a similar theory in which compressibility and heat conduction effects were considered as well. The resulting flows are of a fascinating multiplicity. As a rule, they are steady and when the smooth one-celled flow is no longer possible, other multi-celled flows appear instead. Depending on the primary swirling flow, secondary flows in the end-wall boundary layer may be set up, which can lead to the formation of a circular separation line in the surface. It is typical of such flows that the resulting separation surface can take the form of a circular cylinder and can extend right up the whole tube as far as the opposite end. As a result, the axial and radial velocity components can pass through zero at the separation surface, whereas the swirl component may be continuous in direction. In this way, an inner cell with reversed axial flow may be produced and the process may be continued and lead to multi-celled vortices. Their properties are explored in many ways, especially when they occur in flows involving different media at different temperatures.

It is clear that these end-wall effects and the formation of multi-celled vortices have many applications other than in vortex tubes, especially in atmospheric phenomena, so that there is a wide field for research. Somewhat related is the work of Graebel & Richards* who reported on their theoretical and experimental studies of the flow due to a rotating disk in an infinite fluid with a sink at the centre of the disk. They found, in particular, the location of the streamline on the disk which divides the flow into the sink from the outward flow. Again related to these are the problems of the bath-tub vortex and of the intake vortices. The latter were observed experimentally by Norbury* who found that there was a pair of counter-rotating vortices if external disturbances were excluded. Otherwise, there could be one vortex or none at all. This would suggest that the phenomenon may in some cases involve a separation of the flow from the surface beneath the intake in the form of a vortex sheet rather than separation from a singular point; the sheet may then roll up along its free edges and form two cores.

The occurrence of columnar vortices in rotating fluid systems

In most of the vortex motions discussed so far, it is quite clear how the vorticity is generated and where it comes from. This is not so in many rotating fluid systems, especially in the atmosphere, and the appearance of columnar vortices there presents a major problem. Vortex motions in the atmosphere offer a bewildering variety of particular types and also of scales, from cyclonic circulations thousands of kilometres in diameter through tornadoes and hurricanes to fire whirls and dust devils of diameters of the order of 10 m, and to micro-scale vortices which may be regarded as elements of turbulent motions. The latter were identified by some extremely careful experimental work by Sakagami* as long vortex columns carried along by the wind, with nearly constant circulation outside them.

The mechanisms by which vortex columns of larger scale are created are as yet only incompletely understood. In a few instances, vortex-sheet separations from islands or sharp ridges have been observed from satellites, as reported by Riley & Stroud*. Like trailing vortices behind aeroplanes, these may persist without much diffusion of vorticity for very long distances (of the order of several kilometres in the case of aeroplanes and much more in meteorological cases). In most other instances, however, other explanations are being sought. One possible mechanism involves buoyancy-driven vortices. The action of localized vertical thermal convection currents may lead to a flow field of horizontal convergence. In the presence of an initial not-quite-random rotation of one predominant sense, the conservation of angular momentum may then lead to the development of a strong vortex column at the centre of convergence. Solutions of the Navier–Stokes equations involving thermal convection were presented by Kuo* and discussed in terms of atmospheric vortices. Converging flow patterns with rotation were analysed by Businger, Bergman & Turner* and supplemented by observations of dust devils. The difficulties involved in observing these phenomena (although satellite observations present a new tool of research) are reflected in the fact that it is not yet known in what circumstances the axial flow along the core is upward, or downward, or both in multi-celled systems; and what part, if any, is played by cloud formations and other conditions above the column, by boundary-layer phenomena on the ground, and by the boundary conditions at large distances from the centre. Morton* reported on a set of model tests in which turbulent jets of buoyant liquid were released along the axis of a tank of water initially in a state of rigid-body rotation. He found that the relative sense of the buoyancy of the jet could have profound effects on the changes in the distribution of vorticity.

There may also exist other mechanisms for producing columns of concentrated vorticity; some have been observed or are tentatively suggested. Fultz* reported on tests with a rotating rectangular tank containing liquid with a free surface. He discovered that an intense vortex near the axis could be generated when the axis of rotation was slightly tilted away from the vertical. A new theory by Scorer* involves a stirring mechanism by the random occurrence of strong local thermal convection currents. Scorer argued that the stirred fluid will tend to acquire zero vorticity so that, if the whole system were originally in a state of uniform rotation, the conservation of angular momentum will lead to the concentration of the vorticity near the centre of the system. The tropopause would then take the shape of the surface of the bath-tub vortex and a steady motion like that in a hurricane could become established.

The appearance and structure of vortex wakes

The last group of papers was concerned with the vortices shed into the wake of bodies, primarily in flows past circular cylinders, and the often observed periodic occurrence of regions of high vorticity, as in Kármán vortex streets. This classical but lively subject was reviewed by Wille* in an invited lecture (see also Morkovin 1964). Among some possibilities for future work it was suggested that the flow near the body itself would repay further study. Some reference to both big-scale and small-scale periodicity in the shedding of vorticity has already been made above. The occurrence of subsidiary regions of high vorticity along vortex sheets can be explained on the basis of inviscid flow, as has been done by Abernathy & Kronauer (1962) and, at this symposium, by van de Vooren*. Kronauer* and Timme* investigated further the transport of vorticity into these regions. Kronauer regarded the frequency as determined by the need to satisfy a feedback condition, i.e. the effect of the wake on the conditions near the cylinder must be such as to produce the vorticity shedding which in turn produces the wake. By a kinematic analysis, he found that the growth of the wake culminates in the formation of vortices of fixed strength and that the disturbance wave-number is sharply set and of the right order. He was thus able to provide a more fundamental explanation of the shedding mechanism than that given earlier by Roshko (1961). Timme* presented experimental results consistent with this theory. He also showed that viscosity can be ignored in the generation and growth of the vortices and that, according to the work by Michalke*, even the instability and the final disruption of the vortices, leading to general turbulence, were governed mainly by the ratio between the circulation of an individual vortex and that about the portion of the shear layer between adjacent vortices. The Reynolds number thus comes in only as far as the flow past the body itself and the occurrence of the initial separation are concerned. Of course, the above results will not hold at very low Reynolds numbers. In general, it appears to be justified and possibly helpful in many cases to regard the regions of high vorticity as cores of rolled-up vortex sheets. Gerrard* noted that the strengths of these individual cores in the wake were only a fraction of the circulation shed in one period from the boundary-layer separation point and proposed two possible explanations of this phenomenon.

Calculations of viscous, two-dimensional, time-dependent wake flows were presented by Fromm*. The flow was typically that past a thin slab of rectangular section set normal to a uniform flow in a channel; the Reynolds number based on the breadth of the plate could be as high as 6000; effects of heat transfer were included. The results of these 'numerical experiments' were presented most impressively in the form of films of the development of the motion with time; streamlines, streaklines, vorticity contours and isotherms were shown in this fashion. Although the asymmetrical wakes shown developed from introduced disturbances, similar asymmetries were found to arise eventually from entirely symmetrical boundary conditions. It would be of great interest to refine this numerical technique in the near field of the body and, if it were possible, to extend the technique to cover the predominantly inviscid flows at higher Reynolds numbers still.

Uberoi* presented some experimental results on the formation and properties of the vortex wake behind a heated circular cylinder. He found that with increasing cylinder temperature, the critical Reynolds number for the cylinder itself increases and that the Strouhal number of the periodic vortex shedding decreases.

Several investigations were concerned with the detailed structure of the wake. Wehrmann* reported on a new hot-wire technique to measure two components of the velocity fluctuations. This made it possible to determine the vortex strength, and the correlation of velocity and pressure signals accorded with the hypothesis of a moving system of vortices. In view of the widespread use of flow visualization techniques, it is also important to understand the relation between the motion of fluid particles, the transport of other matter and the motion of the vortices. Michalke* calculated the shape of liquid lines formed by the same fluid particles, at different times. He found that an initially straight line of fluid particles positioned inside a free shear layer showed indeed a tendency to roll up. Berger* investigated Kármán vortex streets by analysing hot-wire measurements and concluded that there were indications of an apparent discrepancy between the experimentally determined values of the carrier, or phase, velocity of the vortices and the velocity of the vortex centres. This might be resolved by taking account of the fact that the flows considered were not plane but three-dimensional, such as those past two-dimensional circular cylinders where the vorticity is shed slantwise, as discussed above. Other flows of this kind are those behind tapered cylinders and whirling cylinders described by Taneda*; again, the vortex filaments were found to be straight but inclined to the axis of the cylinder.

Taneda* presented observations of various approximately two-dimensional flows behind circular cylinders in a water tank, which were made visible by the skilful use of aluminium lamellae and condensed milk, and which shed considerable light on the flow phenomena. He observed stable periodic concentrations of vorticity above Reynolds numbers of about 30. This critical Reynolds number is raised to about 120 when the cylinder is placed between two walls 1·5 cylinder diameters apart. Taneda further found that both laminar and turbulent wakes may lead to periodic wake configurations, like a Kármán street, but that they always break down into highly turbulent motions at some distance downstream. This breakdown phenomenon is as yet very little understood; no connexion has yet been established with the breakdown of streamwise three-dimensional vortex cores discussed above. This turbulent part of the wake may, however, rearrange itself to form another periodic wake with asymmetric concentrations of vorticity, and Taneda found that this process could repeat itself over and over again until the wake became too diffuse for observation. Somewhat similar observations were reported by Birkhoff & Eckermann* from hypersonic wakes behind cones at a Mach number of about 15, where the wake remains aperiodic for a considerable distance behind the body but then assumes a spiral or bifilar structure. The latter phenomena remain so far unexplained (see also Fay & Goldburg 1963).

Tani* presented results for circular cylinders at higher Reynolds numbers in the range from 3×10^4 to 10^6, which covers the well-known critical Reynolds number where the drag coefficient falls. He found no periodic shedding at low supercritical Reynolds numbers, when transition occurs through relatively short separation bubbles on the body. At the high supercritical Reynolds numbers for which the drag coefficient is again high, and which Tani called transcritical, periodic vortex shedding occurred again and was of the same nature as at subcritical Reynolds numbers, with the Strouhal number increased by about 40%. Thus there appears to be a close connexion between the wake structure and the near-flow pattern over the body itself and this suggests an attempt to find a relation between the wake pattern and the drag force on the body. Imai* presented the outline of a new theory of this kind which may also explain the formation of the secondary and higher Kármán vortex streets further downstream, which Taneda observed.

Sato* reported on his studies of wakes behind thin flat plates normal to the stream and distinguished three main regions: a laminar region immediately behind the plate without detectable velocity fluctuations; a second, linear, region with small sinusoidal velocity fluctuations whose amplitude and phase distributions are in good agreement with the predictions of linearized stability theory; and, after some transition, a third, non-linear, region with large sinusoidal velocity fluctuations partly of the same and partly of double the frequency of those in the linear region, indicating the existence of a double row of vortices. Sato supplemented his experimental work with a theoretical analysis with promising results, not only in the linear region but also in the non-linear region where non-linear terms representing Reynolds stresses must be retained in the equations of motion. Thus some features of the flow past flat plates, where the location of the separation lines is firmly fixed were clarified considerably.

Most of the work just described was concerned with flows which were at least nominally two-dimensional. An essentially three-dimensional flow was described by Kendall* who presented the results of his investigations of the flow past spheres. Hot-wire probes indicated that the vortex configuration was probably a chain of loops for $\mathrm{Re} < 400$ and turbulent with periodic fluctuations for $400 < \mathrm{Re} < 40{,}000$. At $\mathrm{Re} = 2000$, the periodic pattern was nearly axisymmetric and of single-pitch helix-like form with a preferred rotary direction. But the time-average circumferential velocity across a wake diameter indicated that the circulation about the wake axis was small, suggesting that a further vortex filament of the opposite sense occurred, possibly along the axis of the helix. The sphere thus experienced at most a small torque. The vortex loops and helical patterns could be clearly seen in flow-visualization studies. Other work on three-dimensional wakes behind spheres has been reported by Magarvey & Bishop (1961).

Further work, again mostly on circular cylinders, is concerned with the effects on the wake flow of time-dependent boundaries. Fiszdon* examined the effects of periodic motions of the body in the fore-and-aft direction, obtaining solutions in the form of expansions in series for very low Reynolds number. Taneda* reported that periodic wakes were produced experimentally from a circular cylinder forced to oscillate laterally at a Reynolds number as low as 1 rather than about 30, as in the case of a fixed cylinder. Also, by an appropriate periodic disturbance, stable symmetrical vortex streets could be produced, a configuration which has never been observed in natural flow. Dosanjh* had passed a normal shock front over a circular cylinder and observed a Kármán vortex street with a different spacing from the one expected in steady flow and a rapid breakdown of the vortex cores. Willmarth* reported studies of the motions and wakes of freely falling circular disks, where he found that the diverse motions of the disks exhibited a systematic dependence on the Reynolds number and on the non-dimensional moment of inertia. A single curve in the plane of Reynolds number and moment of inertia separated the régimes of stable and unstable pitching oscillations. At the higher Reynolds numbers the disks performed tumbling motions with periodic pitching and translational oscillations. Some of the significant states could be related to known changes in the wake flow past fixed disks. The laminar wake behind certain of the oscillating disks consisted of two staggered rows of regularly spaced vortex rings.

Comments on the symposium

To obtain some guidance on how effective the symposium had been, the Committee sought written comments from the participants after the meeting. The replies, from a majority, have been of considerable benefit. No attempt will be made here to evaluate these comments in detail, but the impression conveyed is that the cordial hospitality and local organization provided by the Department of Aeronautics and Astronautics of the University of Michigan contributed much to the success of the symposium. Generous financial support was provided by a number of United States agencies, viz. National Aeronautics and Space Administration, Air Force Office of Scientific Research, Advanced Research Projects Agency, Office of Naval Research, National Science Foundation, and Institute of Sciences and Technology (University of Michigan).

A further impression gained from participants is that the communication and exchange of ideas and the opportunity to meet people with common interests were regarded as important benefits resulting from the meeting; together with the stimuli which come from exposure to the efforts of other people and to related work in other fields, and from the opportunity to see and examine a subject from its many angles and to attend to aspects which one would otherwise have neglected. The fact that the subject of the meeting was based on a fundamental physical concept rather than on some engineering application was regarded as helpful; this also prevented the intrusion of commercialism. A welcome feature of the symposium was that participants could understand a large proportion of the lectures and discussions. This undoubtedly distinguished the symposium from many other meetings and was a consequence, not wholly anticipated perhaps, of the rather classical nature of the subject. The view was widely held that it is essential to provide much time for discussions, especially for informal discussions outside the lecture room during breaks. In this connexion, it is worth putting on record the view, derived from experience at this symposium, that the salient points of most pieces of work can be communicated effectively in as short a time as 15 min, if the speaker is carefully prepared.

It is hoped that these notes will enable a reader who was not at the symposium to form an impression of the work presented there and that participants will be reminded of it and be able to follow up particular lines. It is also hoped that the outcome of the symposium will manifest itself in new approaches and ideas in future research.

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